Math 10A with Professor Stankova
Worksheet, Discussion #26; Wednesday, 10/25/2017
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Review

Example

1. Find the solution to $\frac{dx}{dy} = e^{x-y}$ with $x(0) = 0$.

Solution: We can split e^{x-y} as $e^x \cdot e^{-y}$ and now splitting gives

$$e^{-x}dx = e^{-y}dy \implies -e^{-x} = -e^{-y} + C$$

so $e^{-x} = e^{-y} + C$ and $-x = \ln(e^{-y} + C)$ and $x = -\ln(e^{-y} + C)$. Plugging in $x(0) = 0$ gives $0 = -\ln(1 + C)$ so $1 + C = 0$ and $C = -1$. So

$$x = -\ln(e^{-y} - 1).$$

Problems

2. **TRUE** False In order to justify integration by parts, you need the product rule.

Solution: By product rule, we have that $(uv)' = u'v + uv'$ and hence

$$\int uv' = \int udv = \int (uv)' + u'v = \int (uv)' + \int vdu = uv + \int vdu.$$

3. Calculate $\int_0^1 e^{-x}dx$. State the reasoning behind each step.

Solution: An anti-derivative of e^x is $-e^{-x}$. So, by the first fundamental theorem of calculus, we have that

$$\int_0^1 e^{-x}dx = -e^{-x}|_0^1 = -e^{-1} - (-e^0) = 1 - e^{-1}.$$

4. Write an antiderivative of e^{x^2}. State any reasoning why.

Solution: An example is $\int_0^x e^{t^2} dt$. This is an antiderivative because by the second fundamental theorem of calculus, the derivative of this function is e^{x^2}, and hence it is an antiderivative.

5. Find an antiderivative of $f'(x)$. Is it the only one?

Solution: An antiderivative is $f(x)$. This is not the only one because it could differ by a constant, like $f(x) + 5$ is one as well.

6. If $y_1(x)$ and $y_2(x)$ are solutions do $\frac{dy}{dx} = 5y$, show that $y_1 + y_2$ is a solution and explain all steps.

Solution: Since both y_1 and y_2 are solutions, we know that $y_i' = 5y_i$. Therefore, we have that

$$\frac{d}{dx}(y_1 + y_2) = \frac{d}{dx}(y_1) + \frac{d}{dx}(y_2)$$

by the addition differentiation law, and then

$$= 5y_1 + 5y_2 = 5(y_1 + y_2)$$

since y_1, y_2 are solutions. Thus, this shows that $y_1 + y_2$ is a solution, since it satisfies the differential equation.